International Journal of Theoretical Physics, Vol. 33, No. 1, 1994

A Random-Walk Approach to Interference

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Received July 22, 1993

In this paper we sketch a probabilistic particle approach requiring no separate concept of wave to obtain interference. We describe in some detail how things work from a physical standpoint and show with a number of figures how the standard wave concepts are developed from purely particle random walks. For the wave concepts we have in each case a matching probability concept. The preliminary theory developed here is qualitative and stresses the physical character of the assumptions. In particular, we show that the periodic behavior of light is derived from the source and not from individual photons.

1. INTRODUCTION

It is a familiar and classical aspect of the history of quantum mechanics that from the beginning it has been felt that the complementarity of particles and waves must be accepted and used throughout the theory. Depending upon the particular experiment, light rays are to be thought of as either particles or waves. A typical, but important quotation from the early days is the following one from Heisenberg.

> From these experiments it is seen that both matter and radiation possess a remarkable duality of character, as they sometimes exhibit the properties of waves, at other times those of particles. Now it is obvious that a thing cannot be a form of wave motion and composed of particles at the same time—the two concepts are too different . . . The solution of the difficulty is that the two mental pictures which experiments lead us to form—the one of particles, the other of waves—are both incomplete and have only the validity of analogies which are accurate only in limiting cases. (Heisenberg, 1930, p. 10)

The purpose of this paper is to develop a probabilistic particle approach requiring no separate concept of wave as an approach to

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0020-7748/94/0100-0179$07.00/0

interference. The key idea is to use a random walk from which can be derived as a limit a standard wave equation, in particular, the scalar Huygens–Helmholtz wave equation, but the mathematical details of the derivation cannot be presented here. We describe in some detail how things work from a physical standpoint and show with a number of figures how the standard wave concepts are developed in finite approximation from purely particle random walks. One feature that we shall try to stress is that for the standard wave concepts we have in each case a matching probability concept. It need hardly be emphasized, but is still worth noting because of its importance, that we are operating within a framework that takes us outside of standard quantum mechanics, because we are assuming trajectories for photons, as given by the random walks we characterize in the next section. Some readers will be familiar with the idea of trajectories for particles from the approach to quantum mechanics via stochastic mechanics by Edward Nelson and others. The important difference here is that we do not use a random walk that characterizes a standard diffusion from which we can derive the diffusion equation of Brownian motion, but rather use a random walk from which we can derive a wave equation instead. In our judgement it is this critical change in approach that makes possible a thoroughgoing particle explanation of interference as in, for example, the classical two-slit experiment. It is fair to say that our inspiration for this paper was the derivation of the telegrapher equation in Orsingher (1986) from a random walk different from the one used here.

2. THE RANDOM WALK

We restrict ourselves to two spatial dimensions, because it simplifies the presentation, but there is no essential change in going from two to three dimensions. We do formulate ideas in such a way that the theory is relativistically covariant. We list six assumptions about the random walk of photons. (Extension of the analysis to electrons will be considered in a later paper.)

1. A photon is emitted from a source with an expected direction in the plane which we characterize by an angle θ from a fixed axis.
2. This expected direction θ of emission constitutes a state for the photon. In the present analysis, the state of θ of a photon does not change in the absence of charges.
3. What the photon executes is then a random walk parallel to the x_θ and y_θ axes, each at a 45° angle from the direction state θ. The random walk is characterized by going only forward, which sharply differentiates it from diffusion random walks. At each step the photon has

a probability 1/2 of going one step forward parallel to the y axis and probability 1/2 of going one step forward parallel to the x axis.

4. The point source S has periodic emission of photons uniformly in all directions, which gives us a natural circular symmetry. Later we spell out details of the periodic emission.
5. Here we apply the random-walk model to monochromatic light of a fixed energy only, so we do not need to introduce a separate parameter of energy for photons as we would in other cases.
6. The velocity of a photon in each local step in the random walk is $\sqrt{2}c$. As we shall see in a moment, it easily follows from this that the velocity of each mean distribution normal to the direction θ, moving from n to $n+1$ steps in direction θ, has velocity c. [Feynman (1985, pp. 87–90) has a good discussion of this point, i.e., of photons moving locally at a speed greater than c, in his popular book on quantum electrodynamics.]

Figure 1 shows particles in a given direction state θ and what their distribution is after a certain number of steps. Line segments $D1$ shows the distribution on three points with probability 1/4 for each of the endpoints and probability 1/2 for the middle point. Notice how simple these calculations are. They are just standard calculations for a binomial distribution with $p = 1/2$. Line segment $D2$ shows a distribution of four points for $n = 3$; all photons leaving the point source with direction state θ lie on one of these four points after three steps and the probability for each of these

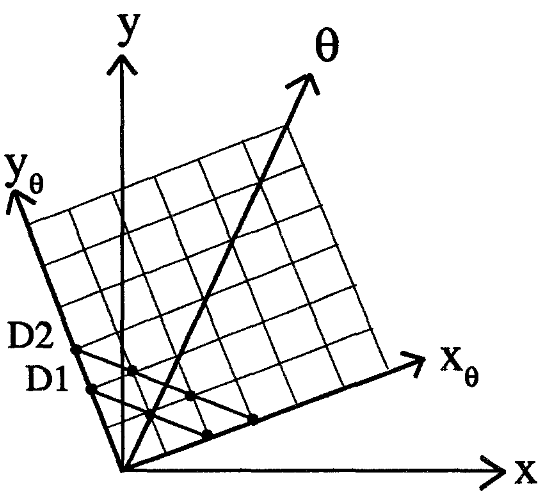

Fig. 1. Mean distribution of photons after two steps ($D1$) and after three steps ($D2$).

four points is determined immediately from the standard binomial distribution, namely, the two interior points each have probability 1/3 and the two endpoints have probability 1/6. As the number of steps increases, it is obvious from the definition of the random walk, and also from Fig. 1, that the variance of the distribution lying on a line normal to the direction state increases without limit. On the other hand, and a critical point for having the properties of waves reflected in the probability distribution, the variance in the direction state θ of the mean distribution of photons after n steps when they have been emitted from a point source is zero, because they all lie on a line segment which is normal to the direction θ and whose endpoints lie on the x_θ and y_θ axes. We call these line segments *distribution domains*. We think of the distribution domains, together with the probability distributions on them, as wavefronts. It is easy to see that these "wavefronts" have velocity c from the characterization of the random walk by computing the distance in the direction θ between two successive distribution domains.

This can also be seen in Fig. 2, which shows not the entire distribution, but a conditional part of a distribution, something important as we pass through barriers as we do in the double-slit experiment. In the case of Fig. 2 we can generalize from having a point source to the usual recursive relation for n steps; in other words, if we have a certain conditional

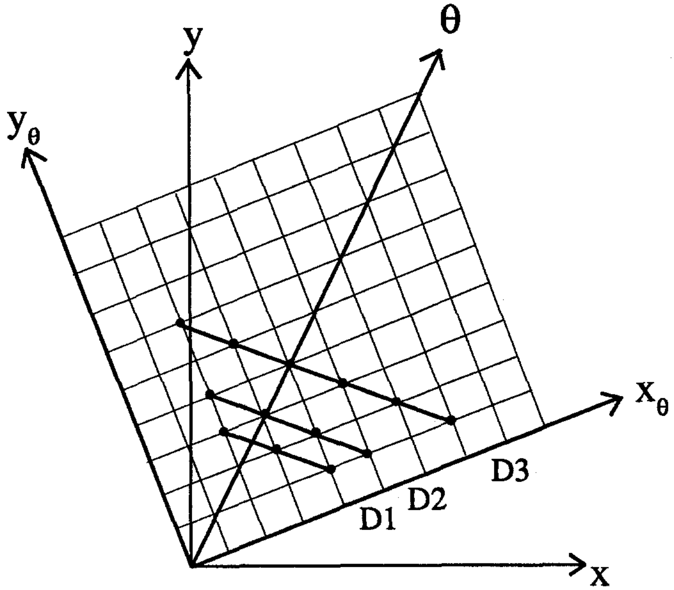

Fig. 2. Conditional distributions of photons.

distribution after n steps we can then compute the conditional distribution of photons after $n + m$ steps. Thus, for example, line segment $D1$ reflects a conditional distribution after $n = 5$, line segment $D2$ with $m = 1$, line segment $D3$ with $m = 3$.

In Fig. 3 we can see how the random walk approximates a spherical wavefront. We have shown the distribution domains after n steps. The scale of the drawing is such that the difference between the radii of the two circles corresponds to taking just one step. What we have done here is randomize uniformly the direction state θ of the emission of photons so that we have circular symmetry. We emphasize that in Fig. 3 what is shown is the complete distribution for particles being emitted in all directions from a source. If one takes any line segment perpendicular to a radius of the inner circle, that line segment is the distribution domain tangent to the inner circle and with endpoints on the outer circle after n steps for emission in the direction of the radius. Of course, the filled-in area between step n (inner circle) and $n + 1$ (outer circle) is the random-walk finite analog of a spherical wave.

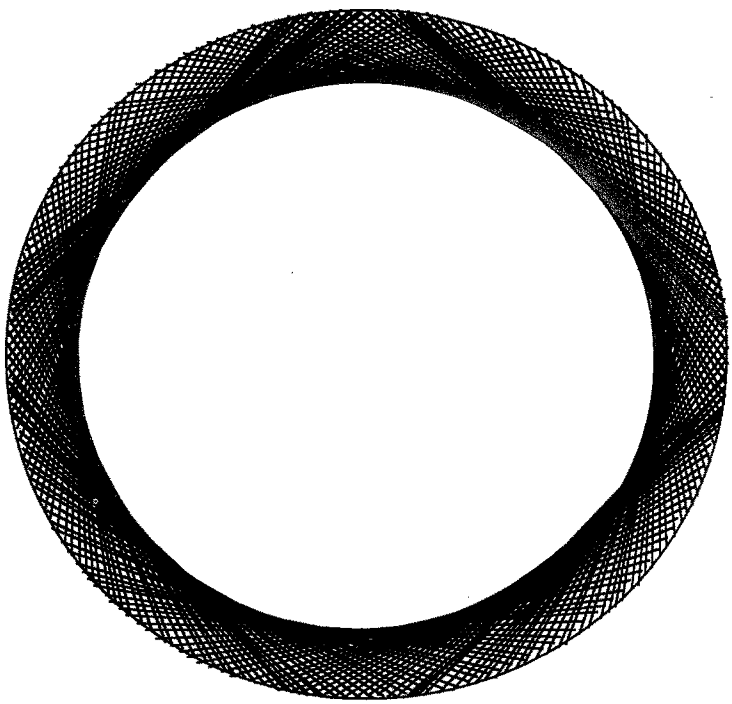

Fig. 3. Distribution domains for uniformly randomized direction state θ.

To emphasize how many different ways conditional distributions arise, we show in Fig. 4 the conditional region in which a photon can move given that it has at step n reached a certain point in the random walk. Roughly put, as the figure makes clear, the domain of possible paths after step n for the particle consists of the positive region bounded on one side by a line parallel to the x_θ axis and on the other side by a perpendicular line parallel to the y_θ axis. In Fig. 4 the boundaries of this region are marked $R1$. We show a similar construction a couple of steps later in the same figure marked as bounding region $R2$.

To complete our physical description of the general situation we need to make some further remarks about the periodic character of the point source for emission of photons. The periodic character of emission is of course a property of the source and not of individual photons. Here we simply follow classical results and assume that the emission follows a cosine periodic function. In particular we assume the following about the source.

1. The source is emitting light by oscillating harmonically, which produces monochromatic light of a given energy. Thus the probability distribution in time for emitting photons in a given direction state follows a sine or cosine law typical of a harmonic oscillator. In particular, we assume a cosine function of the form $C_1(1 + \cos \omega t)$.

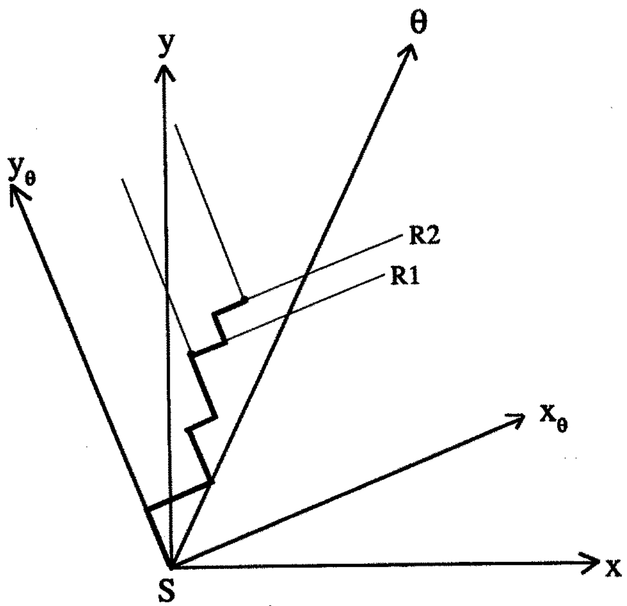

Fig. 4. Conditional regions for photon movement.

2. It is implicit, but we make explicit the assumption that the source is also circularly symmetric in its emission of light.

We know some important properties of the photon emission of the source. First, from the property already mentioned about the zero variance in the direction state, if a distribution domain is at a radial distance d at time t from the source S, then we know its radial distance at time $t + n\Delta t$. This feature is essential for preserving in the various probability distributions the wave form that plays such an important role in the wave theory of light. On the other hand, the variance of the distribution on the distribution domain normal to the direction θ increases as a function of r, the radial distance from the source S. This increase in variance implies a spreading of the particles and thus a decrease in the projection of the wave form on the direction θ of the probability distribution, as shown in Fig. 5.

It is convenient for our later discussion to introduce $p(x, y, t)$ to mean the probability distribution for x and y at time t. This is of course not a probability that characterizes the trajectories, but is just a mean probability at the time t for photons in the x, y plane. A restriction here is that $t > t'$, where t' is the time the photons were emitted from the source. Correspondingly, for a fixed x and y, we get a distribution in t for a finite time interval.

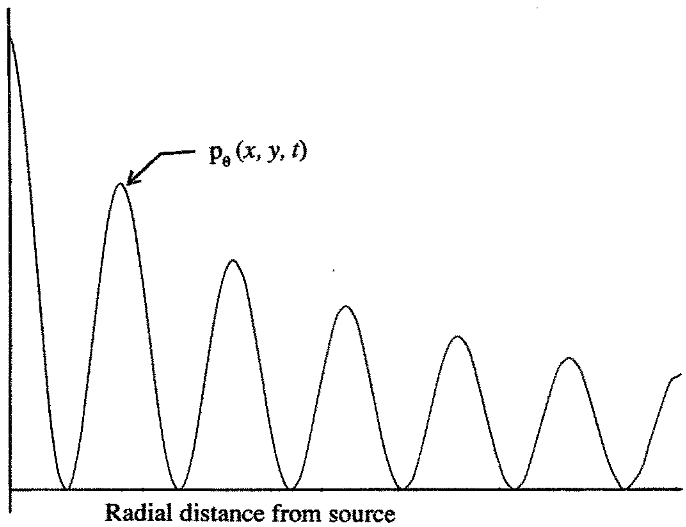

Fig. 5. Cross section of a mean probability distribution with shape of a spreading wave in direction θ.

3. INTERFERENCE

As already indicated, we restrict ourselves to analyzing the classical two-slit experiment showing interference of light. The random-walk analysis of the two-slit experiment is shown in schematic form in Fig. 6. The figure is restricted to showing distribution domains rather than distributions moving in the direction θ. The direction state is at the angle θ with the line perpendicular to the segment between the two slits. The source is located on the perpendicular bisector of this segment. The important things to notice are the succession of plane "waves," each of which is a probability distribution of particles, starting with the domain $D1$, proceeding to $D2$, and then to $D3$. In the case of $D1$, on the left side we can see the left-hand end segment of $D1$ passing through the left slit. From the orientation of the direction of emission we expect most of the particles being emitted in direction state θ to pass through the right-hand slit. It is important to note that some of the particles actually pass through the second slit. Of course, when we emphasize this point remember that any given photon passes through *only* one slit, either the left or right one, but never both. As we move on to $D2$ we can see that the distribution is now broken into two pieces being created by the barrier of the segment between the two slits. So we now have the case of $D2$, a left-hand segment, a space with no particles in the distribution because of the segment barrier between the two slits, and then the right-hand piece of the $D2$ distribution of particles. Finally, with mean distribution $D3$ we have now still two pieces, but one piece is entirely through the slit and the other piece is in front of

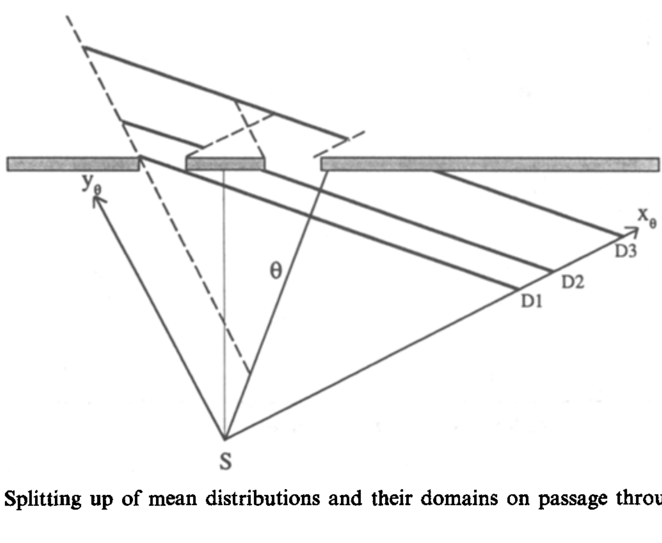

Fig. 6. Splitting up of mean distributions and their domains on passage through slits.

the slit, where all the photons on the second piece on the right will be absorbed by the barrier. What is particularly interesting is that the continuous piece of the distribution, lying beyond the slits after having passed through the slits, shows a small piece of superposition (to use a classical term) arising from the particles passing through the left slit and the particles passing through the right slit.

It is not possible in this short paper to exhibit the detailed computations leading to the distribution on the absorbing screen in the case of the two-slit experiment. What we would like to do in conclusion is to show various aspects of the random-walk approach that are crucial to getting the interference pattern. We emphasize that these arguments do not prove that we will get the full interference pattern. What they do is to give a sense of key sources of the interference from a probabilistic standpoint.

In Fig. 7 we show in the simplest case how we can have for a single fixed direction θ the result that a greater number of particles arrive at P' which is further from the symmetry center of the absorbing screen than point P. Note of course that both P and P' are to the right of the center. The reason for this "local maximum" is obvious. To the left of P is a forbidden region for any particles to arrive that have direction state θ, because the line leading to point P makes a 45° angle with the direction θ, i.e., is the y_θ axis.

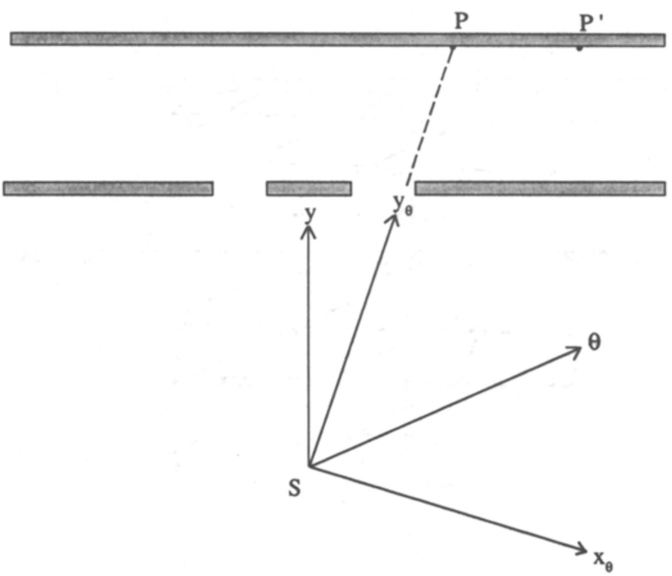

Fig. 7. Local maximum on the screen after passage through a single slit of a single mean distribution.

We now turn to the last point that we have space to discuss. This is how the classical consideration of the periodicity of the source is also used in our random-walk approach to interference. The situation that we will consider here is examining the intensity at a given point P as it is contributed to by two distinct direction states θ_1 and θ_2 of photons. For comparison of the effect we shall, without showing it in Fig. 8, consider a variation in the angle of θ_2 to give us θ_2'. What is familiar in the classical discussions of the two-slit experiment is that θ_2 can be varied so that the number of steps from the source to P is such that the difference in the number of steps between the path with the direction θ_1 and that with the direction θ_2 is such that they are exactly in phase, in the sense of the phase of the periodic emission, on arrival at P. This means for all times t of emission these two direction states of photons are adding to each other, to give, from the standpoint of these two direction states, the maximum intensity at P. Now by slight variation in the angle of θ_2 we get θ_2' such that the difference in the two distances for the two angles of emission of photons is completely out of phase in terms of the periodic cosine function of emission. In this case for these two direction states the intensity at the point P on the screen will be minimal. Notice that this difference is wholly to be accounted for in terms of the distribution of particles, the periodicity of emission, and the difference in number of steps for the two direction states.

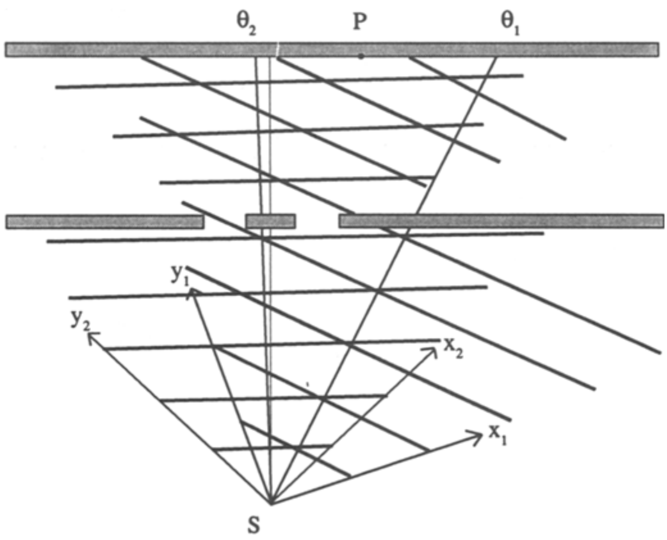

Fig. 8. Interference arising from the path difference of two direction states and the periodic source of emission.

Finally, we emphasize that a distribution and its distribution domain, but not an individual photon, can simultaneously go through both slits, as two separate conditional distributions.

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